

The Crystal Structure of Trisacetylacetonatochromium(III)*

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The crystal structure and molecular configuration of trisacetylacetonatochromium(III), $\text{Cr}(\text{CH}_3\text{COCHCOCH}_3)_3$, has been determined by X-ray diffraction techniques. The structure consists of four discrete molecules linked together by van der Waals forces in a unit cell of $P2_1/c$ symmetry and of dimensions $a_0=14.031$, $b_0=7.551$, $c_0=16.379$ Å, $\beta=99.06^\circ$. Unusually large anisotropic thermal parameters for one of the chelate rings appear to be associated with a lattice wave of large amplitude rather than a slight disordering in the molecular packing along the b direction.

Introduction

Early crystal studies (Astbury, 1926) indicated that the trivalent metal acetylacetonates may be grouped into three isomorphous series. From cell dimensions and space group considerations it appeared that the complexes of aluminum(III), manganese(III), cobalt(III), and chromium(III) and one allotrope of the gallium(III) complex formed a monoclinic isomorphous set. However, the structure determination of the cobalt(III) complex (Padmanabhan, 1958) and preliminary results for the aluminum(III) and chromium(III) complexes (Shkolnikova & Shugam, 1956) differed with respect to the molecular packing. Later Shkolnikova & Shugam (1960) reported the crystal structure determination of the chromium(III) complex with, however, a relatively high discrepancy index ($R=28.8$). Together with Roof's (1956) results on the iron(III) complex, Forman & Orgel (1959) found their infrared data for the chromium, manganese and iron complexes consistent with a Jahn-Teller distortion for the manganese complex. In order to gain an insight to the nature or extent of the distortion, the crystal structure of the manganese complex was determined (Morosin & Brathovde, 1964). During the refinement of the manganese complex it became apparent that an error in the structure determination of the chromium complex had been made by Shkolnikova and Shugam.

Experimental

Standard methods (Bryant & Fernelius, 1957) were used to prepare the chromium complex. The compound was purified by recrystallization and, subsequently, vacuum sublimation. Crystals were grown by slow evaporation at room temperature from an acetone solution.

Equi-inclination Weissenberg photographs were taken with $\text{Cu } K\alpha$ radiation to verify the symmetry of the lattice. The following cell dimensions were obtained from a modified Picker diffractometer ($\lambda\text{Mo } K\alpha_1=$

0.70926 Å) equipped with a G. E. single-crystal orienter and are compared with values determined by Astbury (1926).

		Astbury
a_0	14.031 ± 0.009	14.2
b_0	7.551 ± 0.005	7.62
c_0	16.379 ± 0.011	16.5
β	99.06 ± 0.020	99.13°

Systematic absences of $0k0$ for $k=2n+1$ and of $h0l$ for $l=2n+1$ are consistent with the space group $P2_1/c$. With four molecules per unit cell, the calculated and observed densities are 1.362 and 1.374 g.cm $^{-3}$, respectively.

Intensity data were collected with a Picker diffractometer equipped with a G.E. single-crystal orienter, a scintillation counter, and a molybdenum X-ray tube employing balanced zirconium and yttrium filters. All independent reflections permitted by the space group in the sphere with $\sin \theta/\lambda$ less than 0.5 (2θ less than 45°) were measured by a fixed count (10^n counts with $n \geq 3$, time > 10 seconds) technique. There were 1490 unique reflections of which 224 were considered to be unobserved. No absorption corrections were applied to the intensity data for the spherically ground ($r=0.03$ cm; $\mu=8.5$ cm $^{-1}$) crystal specimen.

Lorentz and polarization factors were applied to the observed data. Structure factors were calculated with Cr (Thomas & Umeda, 1957), C, O, and H (Berghuis, Haanappel, Potters, Loopstra, MacGillavry & Veenendaal, 1955) scattering factors and Cr dispersion corrections (Dauben & Templeton, 1955). Dispersion corrections were included by the method of Templeton (1955). Four reflections, 002, 200, 020, and 110 were found to be considerably affected by secondary extinction and were omitted from all difference and differential syntheses and in the calculation of R . Calculations were carried out on a CDC 1604 computer.

Determination of structure

A comparison of the observed and calculated structure factors listed by Shkolnikova & Shugam for their chrom-

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ium complex with the corresponding ones for the manganese complex was made. This comparison, together with their reported discrepancy index of 0.288, suggested that the crystal structure of the chromium complex is probably more similar to that of the manganese complex than to the molecular packing proposed by Shkolnikova & Shugam. Refinement to about $R=0.30$ for the manganese complex (Morosin & Brat-hovde) with a trial structure similar to Shkolnikova & Shugam's had led to an incorrect determination for that crystal structure.

A three-dimensional Patterson synthesis was calculated. The general features of the synthesis were similar to that of the manganese complex. With the slightly different positional parameter coordinates for the chromium and oxygen, a three-dimensional Fourier synthesis clearly showed the parameter positions of all the carbon atoms although for one chelate ring the electron density distribution at several atomic sites was elliptical rather than spherical. These parameter positions together with individual isotropic temperature factors were refined by several differential syntheses (Booth, 1946), corrected for series termination, to a discrepancy index, R , of 0.11. For the numbering scheme illustrated in Fig. 1, the isotropic thermal parameters for the three carbon atoms [C(12), C(X1) and C(X2)] which appeared with elliptical electron density distributions were large (9.0, 10.8, and 9.4 Å², respectively) compared with the average value of the other carbon atoms (6.4 Å²). A three-dimensional difference synthesis was calculated. Anisotropic motion for several other carbon atoms was evident; therefore, refinement was continued with anisotropic thermal parameters. Inclusion of hydrogen atoms as fixed atom contributions ($B=8.0$) did not significantly change the value of R nor positions of the methyl carbons. Since

there is uncertainty as to the proper coordinates for these hydrogen atoms because of the large anisotropic motion of several methyl groups, the final refined structure (shifts $<0.1\sigma$) does not include any contribution from hydrogen atoms. The final discrepancy index, omitting the four reflections strongly affected by secondary extinction and unobserved reflections, is 0.066 (with no omitted reflections, $R=0.074$).

The final values of the positional and thermal parameters are given in Tables 1 and 2 respectively. Observed and calculated structure factors are listed in Table 3. The molecular packing arrangement is illustrated in Fig. 2. The average estimated standard deviations (e.s.d.) of the positional parameters calculated by the method of Cruickshank (1949) are 0.0014, 0.0065 and

Table 1. Final atomic coordinates

Cr	x	y	z
Cr	0.23771	0.26578	0.47071
O(1)	0.11258	0.33858	0.41152
O(2)	0.18055	0.18920	0.56564
O(3)	0.36548	0.19218	0.52550
O(4)	0.29698	0.34347	0.37596
O(5)	0.21717	0.02765	0.42352
O(6)	0.25741	0.50286	0.51967
C(4)	0.30395	0.25276	0.31081
C(5)	0.23428	-0.02536	0.35347
C(45)	0.27410	0.08116	0.29743
C(X4)	0.34770	0.35068	0.24573
C(X5)	0.21023	-0.21721	0.33426
C(6)	0.33308	0.56527	0.56060
C(3)	0.43110	0.29259	0.56316
C(36)	0.41881	0.47041	0.58000
C(X3)	0.52800	0.20577	0.59184
C(X6)	0.32767	0.75494	0.59008
C(1)	0.03285	0.31731	0.43459
C(2)	0.09222	0.18624	0.57122
C(12)	0.01789	0.23933	0.50880
C(X1)	-0.05339	0.37613	0.37093
C(X2)	0.06938	0.11372	0.65401

Table 2. Anisotropic temperature factors

Anisotropic thermal parameters are of the form $\exp(-\frac{1}{4} \sum_{i=1}^3 \sum_{j=1}^3 B_{ij} h_i h_j a_i^* a_j^*)$.

	B_{11}	B_{22}	B_{33}	B_{12}	B_{13}	B_{23}
Cr	4.794	4.627	3.485	0.748	0.669	-0.138
O(1)	6.246	7.364	5.254	2.132	-0.838	-0.538
O(2)	5.489	7.315	4.509	-0.050	1.249	-0.056
O(3)	5.112	5.123	5.337	0.760	0.681	0.295
O(4)	7.745	5.030	4.577	-0.192	1.543	0.063
O(5)	6.401	4.477	5.157	-0.176	0.646	-0.387
O(6)	6.550	5.001	4.713	1.299	0.573	-0.564
C(4)	5.373	6.081	4.061	0.792	0.357	-0.483
C(5)	4.878	5.262	4.256	1.246	-0.483	-0.800
C(45)	6.342	5.899	5.140	0.353	0.633	-0.474
C(X4)	8.354	8.497	5.255	-1.188	2.248	0.150
C(X5)	7.175	5.133	6.936	-0.319	-0.550	-1.337
C(6)	7.363	5.653	3.895	-0.371	1.399	-0.227
C(3)	5.294	7.309	4.398	0.495	1.606	1.208
C(36)	7.110	6.238	6.306	-0.433	1.562	0.025
C(X3)	4.923	11.476	7.961	1.401	0.515	0.750
C(X6)	11.839	5.213	7.084	-0.225	1.582	-0.347
C(1)	6.240	14.890	6.471	3.017	-0.685	-2.320
C(2)	6.511	13.964	4.819	-1.298	1.934	-0.356
C(12)	5.679	30.443	6.772	2.218	1.368	-1.854
C(X1)	8.150	25.291	9.983	7.838	-3.212	-3.833
C(X2)	10.097	25.953	7.334	-6.253	4.570	0.176

Table 3. Observed and calculated structure factors

h	k	l	observed	calculated	h	k	l	observed	calculated	h	k	l	observed	calculated	h	k	l	observed	calculated	h	k	l	observed	calculated	h	k	l	observed	calculated										
1	0	0	261	261	1	0	0	261	261	1	0	0	261	261	1	0	0	261	261	1	0	0	261	261	1	0	0	261	261	1	0	0	261	261	1	0	0	261	261

0.015 Å for Mn, O, and C, respectively. (The e.s.d. value along the b axis for C(12), C(X1) and C(X2) is 0.017 Å).

Anisotropic thermal motion

Because of the large values* obtained for the anisotropic thermal parameters (Table 2) of several atoms, in particular C(12), C(X1) and C(X2), it was considered desirable to obtain some experimental evidence, if possible, to distinguish between thermal motion and a disordered packing of molecules. Patterns involving streaks between reciprocal lattice points or large isolated diffuse spots or areas on Laue photographs which vary strongly in intensity with temperature are known

* The thermal parameters, B_{ij} , have the units Å², hence the corresponding root mean square (r.m.s.) amplitude of displacement might be better for comparative purposes. B_{ij} 's of 3.5, 7.0, and 30.0 would correspond to r.m.s. amplitudes of 0.21, 0.30, and 0.61 Å, respectively. Simple crystal structures, such as the hexagonal-packed metals of Cd and Zn, show r.m.s. displacements along a and c which approach ratios of 2:1 (International Tables for X-ray Crystallography, 1962).

to indicate the existence of large-amplitude high-frequency thermal waves in the solid (Lonsdale, 1959). Therefore, photographs, employing the Laue technique

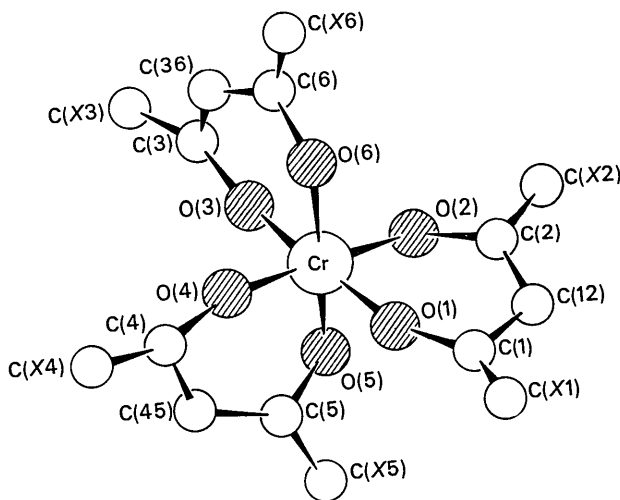


Fig. 1. Labeling scheme for trisacetylacetonatochromium(III).

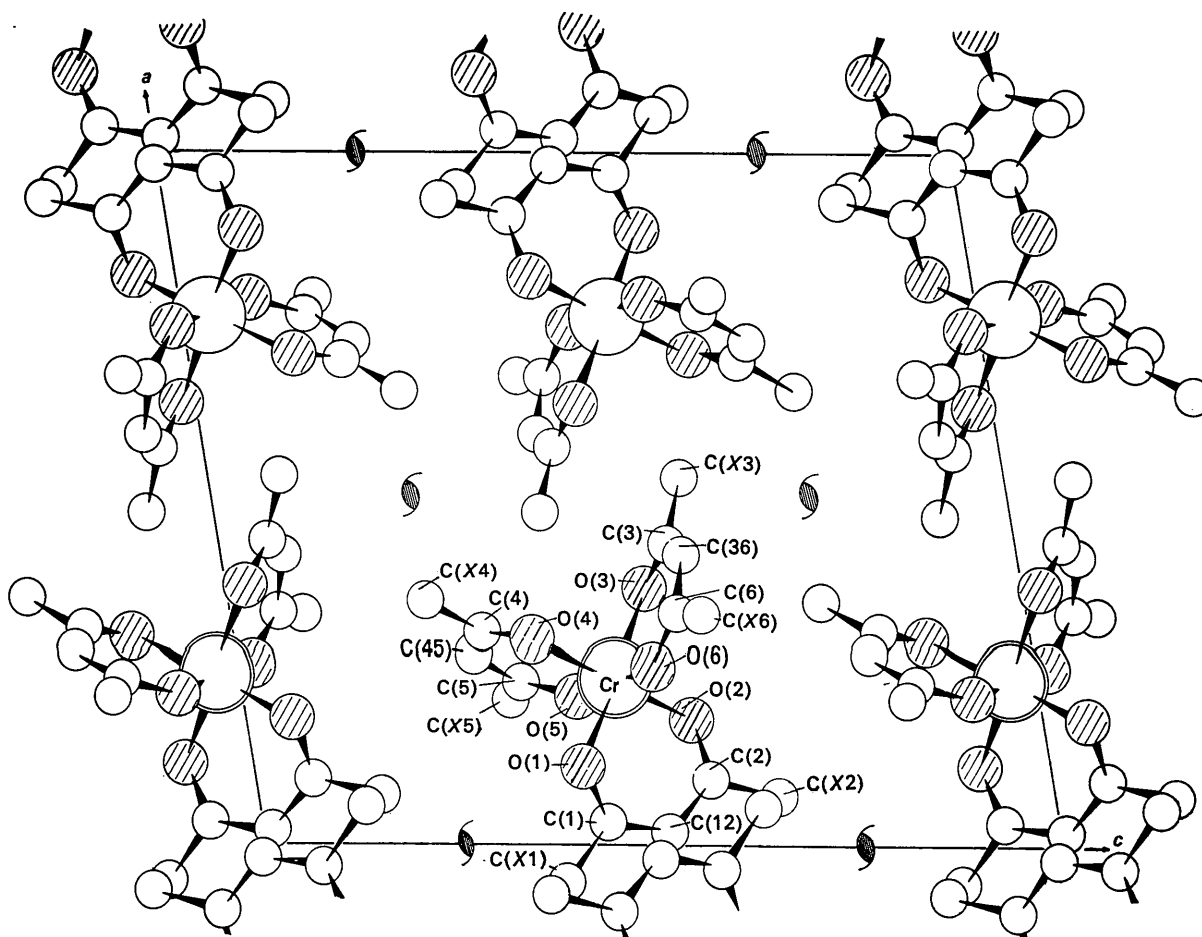


Fig. 2. Molecular arrangement viewed along b_0 axis. Chromium atoms near $y = \frac{1}{4}$ are denoted by double circles; those near $y = \frac{3}{4}$ by a single open circle.

both with unfiltered Mo and filtered Mo $K\alpha$ radiation, were taken at room temperature and in the range -35 to -65°C along the a^* and c^* axes. The well defined patterns of diffuse spots in the room temperature Mo $K\alpha$ photographs disappeared at the lower temperature; however, the patterns were reproduced when the crystal was returned to room temperature. This indicates that the thermal parameters, though large, are probably real and that the 'smearing' of the Fourier syntheses at the atom sites of C(12), C(X1) and C(X2) is due to thermal motion rather than to a disordered packing of the molecules in the lattice.

Discussion

Soon after the structure determination of the manganese(III) complex appeared (to which was appended a note added in proof concerning the chromium(III) complex), the redetermination of the cobalt (III) complex (Pfluger, 1964) and the determination of the rhodium(III) complex (Morrow, 1964) became known to us and established the same molecular packing for all these compounds. These results appear to corroborate the earlier crystal studies of Astbury which indicated these compounds formed a monoclinic isomorphous series.

Tables 4 and 5 list the intramolecular bond lengths and bond angles, respectively, for the molecule (e.s.d. by the method of Cruickshank). There do not appear to be any values which are significantly different from the averages. In Fig. 3, the average values of these intramolecular bond lengths and angles are compared with those for the manganese complex. The average metal-oxygen distances of the two compounds are significantly different (difference $\sim 10\sigma$), and, although the difference between the average values for the C-O bond distances cannot be considered significant, the

C-O bonds of the chromium compound appear to be shorter than those for the manganese compound. The carbon-carbon distances of the two complexes are in agreement with each other. The 1.39 \AA ring C-C bond distance lies between the single-bond (1.54 \AA) and the double-bond (1.33 \AA) values. This partial double bond character is consistent with the expected delocalization of π electrons in the chelate rings.

Table 4. Intramolecular bond lengths

		Average
Cr-O(1)	$1.943 \pm 0.007 \text{ \AA}$	
Cr-O(2)	1.942	
Cr-O(3)	1.951	
Cr-O(4)	1.958	
Cr-O(5)	1.956	
Cr-O(6)	1.959	
		1.951 \AA
O(1)-C(1)	1.244 ± 0.016	
O(2)-C(2)	1.255	
O(3)-C(3)	1.272	
O(4)-C(4)	1.282	
O(5)-C(5)	1.271	
O(6)-C(6)	1.252	
		1.263
C(1)-C(12)	1.393 ± 0.021	
C(2)-C(12)	1.396	
C(3)-C(36)	1.384	
C(6)-C(36)	1.390	
C(4)-C(45)	1.366	
C(5)-C(45)	1.398	
		1.388
C(1)-C(X1)	1.530 ± 0.021	
C(2)-C(X2)	1.539	
C(3)-C(X3)	1.513	
C(4)-C(X4)	1.502	
C(5)-C(X5)	1.506	
C(6)-C(X6)	1.514	
		1.517

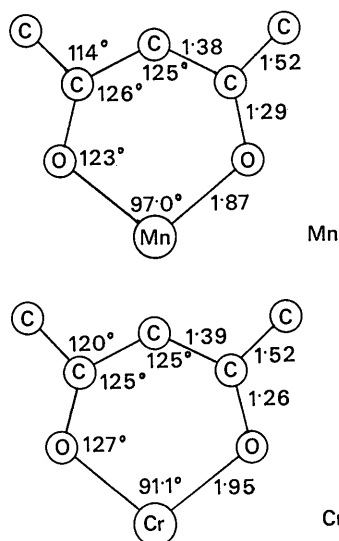


Fig. 3. Average intramolecular bond lengths for the chromium and manganese complexes.

The oxygen-oxygen separations in the octahedron surrounding the chromium ion are listed in Table 6. This octahedron is very much more regular (the 2.786 \AA average intrachelate O-O separation might be considered possibly significantly different from the average interchelate O-O separation of 2.751 \AA) than that found for the manganese(III) complex (the corresponding distances are 2.805 and 2.596 \AA , respectively). Preliminary results on the redetermination of the iron(III) complex (Iball & Morgan, 1964) indicate that this octahedron is also regular. The regular environments about the chromium and iron, but not manganese, complexes are in agreement with the infrared data for the respective compounds (Forman & Orgel).

Fig. 4* best illustrates the large anisotropic motion of one of the chelate rings, in particular carbon atoms C(12), C(X1) and C(X2). The ellipsoids, whose principal axes are proportional to the root mean square thermal displacement along the axes, represent atoms and are drawn to scale with respect to the interatomic

* This figure was drawn at Oak Ridge National Laboratory on a Cal Comp $x-y$ plotter with computer code OR TEP written by Dr C. K. Johnson.

Table 5. Intramolecular bond angles

O(1)-Cr-O(2)	91.66 ± 0.6°	C(1)-C(12)-C(2)	124.06 ± 1.0°
O(3)-Cr-O(6)	90.67	C(3)-C(36)-C(6)	125.61
O(4)-Cr-O(5)	90.95	C(4)-C(45)-C(5)	125.60
Cr-O(1)-C(1)	126.51 ± 0.9	O(1)-C(1)-C(X1)	113.98 ± 1.0
Cr-O(2)-C(2)	126.75	O(2)-C(2)-C(X2)	114.46
Cr-O(3)-C(3)	126.41	O(3)-C(3)-C(X3)	115.92
Cr-O(4)-C(4)	126.49	O(4)-C(4)-C(X4)	114.79
Cr-O(5)-C(5)	127.50	O(5)-C(5)-C(X5)	114.95
Cr-O(6)-C(6)	127.84	O(6)-C(6)-C(X6)	116.36
O(1)-C(1)-C(12)	125.70 ± 1.0	C(12)-C(1)-C(X1)	120.23 ± 1.0
O(2)-C(2)-C(12)	124.87	C(12)-C(2)-C(X2)	120.66
O(3)-C(3)-C(36)	124.69	C(36)-C(3)-C(X3)	119.38
O(4)-C(4)-C(45)	125.19	C(45)-C(4)-C(X4)	120.01
O(5)-C(5)-C(45)	124.14	C(45)-C(5)-C(X5)	120.90
O(6)-C(6)-C(36)	123.81	C(36)-C(6)-C(X6)	119.83

Table 6. Oxygen-oxygen distances found in the octahedron about the metal ion

O(1)-O(2)	2.786 ± 0.013 Å	O(1)-O(5)	2.754 Å
O(3)-O(6)	2.781	O(3)-O(4)	2.731
O(4)-O(5)	2.791	O(2)-O(6)	2.750
O(1)-O(4)	2.735	O(2)-O(3)	2.770
O(4)-O(6)	2.771	O(3)-O(5)	2.746
O(6)-O(1)	2.764	O(5)-O(2)	2.742

separation. Preliminary results on the cobalt(III) and rhodium(III) complexes also indicate anisotropic motion of this particular ring. The packing arrangement of the molecules requires this particular chelate ring to be stacked directly above itself along the *b* axis. Hence, the shorter intermolecular (nearest neighbours) separations (Table 7) for C(X1), C(12) and C(X2) involve the C(X2'), C(12') and C(X1'), respectively, which are translated $\frac{1}{2}$ of a unit cell *b* by the 2₁ axis. This type of molecular packing allows this chelate ring sufficient freedom to oscillate (in unison with its symmetry related members) with a large amplitude. The

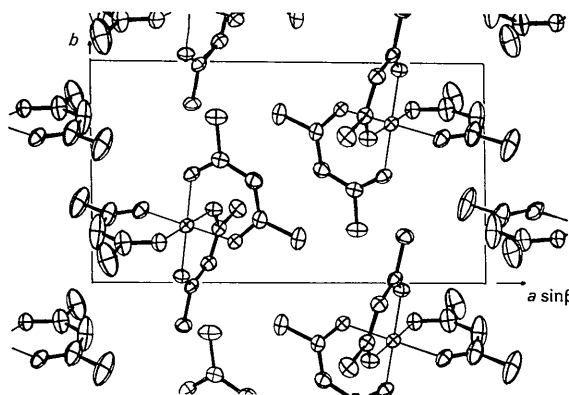


Fig. 4. Perspective view for trisacetylacetonatochromium(III) along c_0 axis. Atoms are represented as ellipsoids with principal axes proportional to the root mean square thermal displacements along the axes. Only two molecules per cell (those contained between $z \approx 0.25$ and 0.75) are included. The second set (z between ~ 0.25 and ~ 0.75) are related to the illustrated set by the *c* glide and their inclusion would only serve to obscure the drawing.

Table 7. Intermolecular (nearest neighbour) distances and angles

Intermolecular distances (Å) for each methyl carbon are followed in parentheses by the C(*i*)-C(X*i*)-C(inter-molecular) angle (°). The C(*i*)-C(X*i*) intramolecular bond distances are shown after each methyl carbon atom. The numbers preceding the atom involved in the intermolecular contacts refer to the following positions:

1 <i>x</i> , 1 + <i>y</i> , <i>z</i>	5 - <i>x</i> , 1 - <i>y</i> , 1 - <i>z</i>	9 <i>x</i> , - $\frac{1}{2}$ - <i>y</i> , $\frac{1}{2}$ + <i>z</i>
2 <i>x</i> , -1 + <i>y</i> , <i>z</i>	6 - <i>x</i> , - <i>y</i> , 1 - <i>z</i>	10 <i>x</i> , $\frac{1}{2}$ - <i>y</i> , - $\frac{1}{2}$ + <i>z</i>
3 1 - <i>x</i> , 1 - <i>y</i> , 1 - <i>z</i>	7 <i>x</i> , $\frac{1}{2}$ - <i>y</i> , $\frac{1}{2}$ + <i>z</i>	11 <i>x</i> , $\frac{1}{2}$ - <i>y</i> , - $\frac{1}{2}$ + <i>z</i>
4 1 - <i>x</i> , - <i>y</i> , 1 - <i>z</i>	8 <i>x</i> , $\frac{1}{2}$ - <i>y</i> , $\frac{1}{2}$ + <i>z</i>	12 <i>x</i> , - $\frac{1}{2}$ - <i>y</i> , - $\frac{1}{2}$ + <i>z</i>
C(X1) 1.530	C(X2) 1.539	C(X3) 1.513
5 C(12) 3.493 (80)	6 C(X1) 3.716 (106)	4 C(X5) 3.676 (153)
6 C(X2) 3.716 (79)	5 C(X1) 3.868 (65)	4 C(45) 3.748 (163)
5 C(X2) 3.868 (113)	7 C(X4) 3.959 (89)	7 C(X4) 3.858 (67)
5 C(X6) 4.112 (119)	7 C(45) 4.109 (95)	4 C(X6) 3.861 (106)
11 C(X2) 4.188 (100)	7 C(X1) 4.178 (156)	3 C(36) 3.880 (77)
5 C(X1) 4.641 (53)	9 C(X5) 4.435 (134)	4 (C(X3) 4.302 (92)
1 C(X5) 4.907 (75)	2 C(X6) 4.762 (75)	2 C(X6) 4.403 (78)
		3 C(X4) 4.449 (96)
C(X4) 1.502	C(X5) 1.506	C(X6) 1.514
11 C(X3) 3.858 (142)	2 O(4) 3.555 (143)	3 C(X3) 3.861 (77)
11 C(36) 3.877 (112)	4 C(X3) 3.676 (74)	8 C(X4) 3.893 (158)
10 C(X6) 3.893 (147)	2 O(1) 3.900 (147)	5 C(X1) 4.112 (85)
11 C(X2) 3.951 (77)	2 C(X4) 4.157 (136)	1 C(X5) 4.251 (76)
1 C(X5) 4.157 (83)	2 C(X6) 4.251 (79)	7 C(45) 4.389 (74)
3 C(X3) 4.449 (96)	12 C(X2) 4.435 (148)	1 C(X3) 4.403 (132)
	11 C(X6) 4.559 (98)	7 C(X5) 4.559 (105)
	2 C(X1) 4.907 (136)	1 C(X2) 4.762 (133)

other two chelate rings are arranged in the crystal in such a manner as to prevent the existence of a similar lattice wave or mode. The reproducibility of the diffuse patterns with temperature cycling together with the packing arrangement suggest the interpretation that the large anisotropic temperature parameters arise from the thermal motion rather than a slight disorder in the molecular packing is correct.

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Diffraction by a One-Dimensionally Disordered Crystal. I. The Intensity Equation

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The intensity equation $I(\varphi) = N \text{spur } \mathbf{V}\mathbf{F} + \sum_{m=1}^{N-1} (N-m) \text{ spur } \mathbf{V}\mathbf{F}\mathbf{Q}^m + \text{conj.}$ for X-rays diffracted by a one-dimensionally disordered crystal has been solved by three methods, *viz.* (i) by using the inverse matrix $(\mathbf{I} - \mathbf{Q})^{-1}$, (ii) by diagonalizing \mathbf{Q} by the similarity transformation $\mathbf{O}\mathbf{Q}\mathbf{O}^{-1}$ and by using the solutions of a characteristic equation $\det(y\mathbf{I} - \mathbf{Q}) = 0$ and a set of simultaneous equations with respect to b_v 's, $\sum_{v=1}^R b_v y_v^m = B_m = \text{spur } \mathbf{V}\mathbf{F}\mathbf{Q}^m$ ($m=0, 1, \dots, R-1$), and (iii) by using B_m and the relation between roots and coefficients of the characteristic equation without solving these equations explicitly. These methods should be applied to the problem only after the order of matrices has been reduced to lower order by considering the symmetry character involved in the matrices.

Explicit expressions are given for three cases, namely the case of different thickness, that of equal thickness, and that of displacement stacking faults. In the case of displacement stacking faults, the three-dimensional Patterson function is given with respect to the distribution of origins of layers. The result is compared with that obtained by Allegra.

Introduction

The intensity of X-rays diffracted by a one-dimensionally disordered crystal such as SiC, CdBr₂, CdI₂, some alloys of the Laves phase, many other alloys and metals showing the stacking faults between cubic and hexa-

gonal close-packed structures, and some kinds of anti-phase domain structures and minerals, has been studied by many researchers, for example, Wilson (1942, 1943), Hendricks & Teller (1942), Zachariasen (1947), Jagodzinski (1949 *a, b, c*, 1954), Méring (1949), Paterson (1952), Kakinoki & Komura (1952; 1954 *a, b*), Gevers